

# Gravitational Waves from Quasi-Circular Black Hole Binaries in Dynamical Chern-Simons Gravity

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Dynamical Chern-Simons gravity cannot be strongly constrained with current experiments because it reduces to General Relativity in the weak-field limit. This theory, however, introduces modifications in the non-linear, dynamical regime, and thus, it could be greatly constrained with gravitational waves from the late inspiral of black hole binaries. We complete the first self-consistent calculation of such gravitational waves in this theory. We find that future gravitational wave detectors could place constraints that are 6 orders of magnitude stronger than current ones.

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*Introduction.*— General Relativity (GR) has been tested to exquisite accuracy in the Solar System and with binary pulsars [1], constraining any possible deviations when the gravitational field is weak and non-dynamical. The non-linear and highly dynamical, *strong-field* regime of GR, however, remains unconstrained. Gravitational waves (GWs) emitted during the late inspiral and merger of compact objects would be ideal probes of this regime. The second-generation of ground-based GW detectors (Ad. LIGO [2], Ad. VIRGO [3], KAGRA [4] and LIGO-India [5]) will allow for the first strong-field tests of GR.

Modified gravity theories that reduce exactly to GR in the weak-field, yet deviate in the strong-field, exist. One example is dynamical Chern-Simons (CS) gravity [6, 7], where the Einstein-Hilbert action is modified by the product of a dynamical scalar field and the Pontryagin topological invariant. Such a correction is unavoidable in superstring theory to cancel certain anomalies [8–10] and it also arises naturally in loop quantum gravity, upon the scalarization of the Barbero-Immirzi parameter [11, 12], and in effective field theories of inflation [13].

Dynamical CS gravity has not been heavily constrained by experiments because (i) it only interacts with gravity and (ii) it reduces exactly to GR in the weak-field limit. This theory has the same post-Newtonian parameters as GR in the weak-field [14, 15]. The leading-order non-vanishing modification to the motion of bodies enters through corrections to Lense-Thirring precession [16]. LAGEOS [17] and Gravity Probe B [18] can thus constrain the theory, but only extremely weakly.

GWs emitted during the late inspiral and coalescence of black hole (BH) binaries encode invaluable information about the fundamental gravitational interaction in the strong field, precisely where CS deviations are largest. Supermassive BH mergers are not ideal for such tests because their radius of curvature is large, and thus, CS corrections are naturally suppressed [19, 20]. On the other hand, stellar-mass BHs, the targets of ground-based GW detectors, are ideal for testing CS gravity because their radius of curvature is small.

Future GW tests of dynamical CS gravity require the calculation of waveform templates with which to filter GW data [21]. This is essential because the signal-to-noise ratio (SNR) of ground-based detectors is expected to be low, at least initially. Therefore, the calculation of templates in dynamical CS gravity is urgent, if we are to test this theory with future GW observations. In calculating such templates, we will discover how a BH binary shrinks due to the loss of energy to GW and scalar radiation, how the binary’s binding energy is modified due to the presence of a scalar field and how Kepler’s third law is corrected. These results are relevant to other astrophysical tests of dynamical CS gravity, for example with low-mass x-ray binaries [22].

*Dynamical Chern-Simons Gravity.*— This theory is defined by the action [7]

$$S = \int d^4x \sqrt{-g} \left( \kappa_g R + \frac{\alpha}{4} \vartheta R_{\nu\mu\rho\sigma} {}^* R^{\mu\nu\rho\sigma} - \frac{\beta}{2} \nabla_\mu \vartheta \nabla^\mu \vartheta + \mathcal{L}_{\text{mat}} \right), \quad (1)$$

where  $\kappa_g \equiv (16\pi G)^{-1}$ ,  $g$  is the determinant of the metric  $g_{\mu\nu}$ ,  $R_{\mu\nu\delta\sigma}$  and  ${}^* R^{\mu\nu\rho\sigma}$  are the Riemann tensor and its dual,  $R$  is the Ricci scalar,  $\vartheta$  is a dynamical field,  $(\alpha, \beta)$  are coupling constants and  $\mathcal{L}_{\text{mat}}$  is the matter Lagrangian density. We define the dimensionless parameter  $\zeta \equiv \xi/m^4$ , where  $\xi \equiv \alpha^2/(\kappa_g\beta)$  and  $m$  is the total mass of the system. The characteristic length scale of the theory is given by  $\xi^{1/4}$  and Solar System tests require  $\xi^{1/4} \leq \mathcal{O}(10^8)$  km [16] (see [7] for more details).

*Adiabatic Quasi-Circular BH Inspirals.*— The inspiral of comparable-mass compact objects can be studied within post-Newtonian (PN) theory, where one assumes all characteristic velocities are much smaller than the speed of light and gravitational fields are weak [23]. Such an approximation at the 3rd post-Newtonian order has been shown to be accurate until the very last stages of the inspiral [24–26]. We here concentrate on quasi-circular orbits, because, by the time GWs emitted in generic orbits enter the sensitivity band of ground-based detectors, they will have circularized due to GW emission [27, 28].

This also holds true in dynamical CS gravity [29], where it has also been shown that spinning BHs exist, albeit in the slow-rotation limit [30, 31].

A circular orbit is fully described by its binding energy  $E$ . In dynamical CS gravity, this quantity contains three contributions: a gravitational potential energy  $E_U$ , a kinetic energy  $E_K$  and a scalar interaction energy  $E_{DD}$ .  $E_U$  can be calculated, to leading PN order, via  $\int U_1' \rho_2' d^3x'$ , where the primes mean that  $U_1$  and  $\rho_2$  are functions of  $x'^i$ ,  $U_A$  is the gravitational potential of BH  $A$  [31]

$$U_A \equiv -\frac{m_A}{r_A} \left( 1 + 3Q_A^{ij} \frac{n_{A\langle ij \rangle}}{r_A^2} \right), \quad (2)$$

and  $\rho_A$  is the density of BH  $A$

$$\rho_A \equiv \left( m_A + Q_A^{ij} \partial_i \partial_j \right) \delta^{(3)}(x^k - x_A^k), \quad (3)$$

with the quadrupole moment given by  $Q_A^{ij} \equiv (201/3584)\zeta(m^4/m_A)\chi_A^2 \hat{S}_A^{\langle i} \hat{S}_A^{j \rangle}$  [31]. Here,  $m_A$  is the individual mass,  $\chi_A = |S_A^i|/m_A$  is the dimensionless Kerr spin parameter,  $\hat{S}_A^i$  is the spin angular momentum unit vector, and  $r_A$  and  $n_A^i$  are the field point distance and unit vector, all relative to the  $A$ th BH, with the angle-brackets representing the symmetric and trace-free operation, i.e.  $n_{A\langle ij \rangle} \equiv n_{Ai} n_{Aj} - (1/3)\delta_{ij} n_{Ak} n_A^k$ . The potential in Eq. (2) can be read directly from the  $(t, t)$  component of the metric of an isolated BH in dynamical CS gravity [31]. The density in Eq. (3) must be calculated by solving  $\square U_A = 4\pi\rho_A$ , with the potential of Eq. (2). Combining all these results,

$$E_U = -\frac{\mu m}{2r_{12}} \left\{ 1 - \frac{201}{1792} \zeta \frac{m^2}{m_1^2} \chi_1^2 \left[ 1 - 3(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)^2 \right] \frac{m^2}{r_{12}^2} \right\} + (1 \leftrightarrow 2), \quad (4)$$

where  $r_{12}$  and  $n_{12}^i$  are the binary's orbital separation and the separation's unit vector,  $\mu = m_1 m_2 / m$  is the reduced mass, and  $(\mathbf{A} \cdot \mathbf{B})$  is the flat-space scalar inner product.

The scalar field has a rest energy and an interaction energy, which is induced because spinning BHs in dynamical CS gravity possess a magnetic-type dipole scalar field. When two such BHs are present, the dipole-dipole interaction energy is

$$E_{DD} = \frac{25}{256} \zeta \frac{m^4}{r_{12}^3} \chi_1 \chi_2 \left[ (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2) - 3(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_2) \right]. \quad (5)$$

This result is derived by analogy with electromagnetically charged BHs in GR [32]. The kinetic energy to leading PN order is  $E_K = \mu v^2/2$ , with  $v$  the relative velocity.

Let us now re-express  $E_U$ ,  $E_{DD}$  and  $E_K$  in terms of  $u \equiv (\pi m f)^{1/3} = (m\omega)^{1/3}$  where  $f$  is the GW frequency and  $\omega$  is the orbital angular velocity. This is achieved by finding the leading-order, effective-one-body, equation of motion for the binary constituents, which in the center

of mass frame is  $\mu r_{12} \omega^2 = F_U + F_{DD}$ . The magnitude of the gravitational and dipole-dipole forces are computed by differentiating the potential and dipole-dipole energies with respect to  $r_{12}$ . We have checked that in the test particle limit, this equation of motion reduces to the modified Kepler's third law found in [31]. From this, we obtain Kepler's third law  $r_{12} = (m/u^2)(1 + \delta C_r u^4)$  with

$$\delta C_r \equiv -\frac{25}{512} \zeta \frac{1}{\eta} \chi_1 \chi_2 \left\langle (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2) - 3(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_2) \right\rangle_\omega - \frac{201}{3584} \zeta \frac{m^2}{m_1^2} \chi_1^2 \left[ 1 - 3 \left\langle (\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)^2 \right\rangle_\omega \right] + (1 \leftrightarrow 2), \quad (6)$$

where we have orbit averaged, as implied by  $\langle \dots \rangle_\omega$ . We also obtain  $v = r_{12} \omega = u(1 + \delta C_r u^4)$ .

We now have all the ingredients to compute the binding energy in the center of mass frame in terms of  $u$ . Combining  $E_U$ ,  $E_{DD}$  and  $E_K$  and taking the orbital average, we find  $E = -(\mu/2)u^2(1 + \delta C_E u^4)$ , with

$$\delta C_E \equiv \frac{25}{256} \zeta \frac{\chi_1 \chi_2}{\eta} \left\langle (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2) - 3(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)(\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_2) \right\rangle_\omega + \frac{201}{1792} \zeta \frac{m^2}{m_1^2} \chi_1^2 \left[ 1 - 3 \left\langle (\mathbf{n}_{12} \cdot \hat{\mathbf{S}}_1)^2 \right\rangle_\omega \right] + (1 \leftrightarrow 2), \quad (7)$$

where  $\eta = \mu/m$  is the symmetric mass ratio. This reduces to Eq. (100) of [31] in the test particle limit.

For a circular orbit, the *balance law* states that the rate of change of the binding energy  $\dot{E}$  must be exactly balanced by the flux of scalar and gravitational radiation taken out to infinity and into any horizons. One can then show that  $\dot{E} = -(32/5)\mu^2 v^4 \omega^2 [1 + \delta C_E u^4] = -(32/5)\eta^2 u^{10} [1 + (\delta C_{\dot{E}} + 4\delta C_r)u^4]$  where [29]

$$\delta C_{\dot{E}} \equiv \frac{25}{19776} \zeta \frac{1}{\eta^2} \left[ \Delta^2 + 2 \langle (\boldsymbol{\Delta} \cdot \hat{\mathbf{v}}_{12})^2 \rangle_\omega \right] + \frac{75}{256} \zeta \frac{\chi_1 \chi_2}{\eta} \left\langle \hat{S}_1^i \hat{S}_2^j (2\hat{v}_{ij}^{12} - 3n_{\langle ij \rangle}^{12}) \right\rangle_\omega. \quad (8)$$

Here,  $\hat{\mathbf{v}}_{12}^i = \hat{v}_1^i - \hat{v}_2^i$  is a unit vector pointing in the direction of the difference of the orbital velocities,  $\hat{v}_{12}^{ij} \equiv \hat{v}_1^i \hat{v}_2^j$ , and  $\Delta^i \equiv (m_2/m)\chi_1 \hat{S}_1^i - (m_1/m)\chi_2 \hat{S}_2^i$ . The first term is a CS correction due to the emission of scalar radiation, while the second one is a CS correction to the emission of gravitational radiation.

*GWs from Quasi-Circular BH Inspirals.*— The balance law allows us to write an evolution equation for the GW frequency:  $\dot{f} = \dot{f}_{\text{GR}}(1 + \delta C u^4)$  and to leading order in the PN approximation  $\dot{f}_{\text{GR}} \equiv (96/5)\pi^{8/3} \mathcal{M}^{5/3} f^{11/3} + \mathcal{O}(u^{13})$  [23] while  $\delta C \equiv \delta C_{\dot{E}} - 3\delta C_E + 4\delta C_r$ , or

$$\delta C = \frac{25}{39552} \zeta \frac{1}{\eta^2} \left[ \Delta^2 + 2 \langle (\boldsymbol{\Delta} \cdot \hat{\mathbf{v}}_{12})^2 \rangle_\omega \right] + \frac{75}{512} \zeta \frac{\chi_1 \chi_2}{\eta} \left\langle \hat{S}_1^i \hat{S}_2^j (2\hat{v}_{ij}^{12} - 3n_{\langle ij \rangle}^{12}) \right\rangle_\omega \quad (9)$$

$$\begin{aligned}
& -\frac{125}{256}\zeta\frac{\chi_1\chi_2}{\eta}\left\langle(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2)-3(\mathbf{n}_{12}\cdot\hat{\mathbf{S}}_1)(\mathbf{n}_{12}\cdot\hat{\mathbf{S}}_2)\right\rangle_\omega \\
& -\frac{1005}{1792}\zeta\frac{m^2}{m_1^2}\chi_1^2\left[1-3\left\langle(\mathbf{n}_{12}\cdot\hat{\mathbf{S}}_1)^2\right\rangle_\omega\right]+(1\leftrightarrow 2).
\end{aligned}$$

This evolution equation then gives the time-domain representation of the GW phase via  $\phi(t) = \int 2\pi(f/\dot{f})df$ .

In the extraction of GWs from noisy data, it is customary to employ the Fourier transform of the GW response function. This quantity can be computed analytically via the stationary phase approximation [33–36]. Neglecting PN amplitude corrections to the GW response, the sky-averaged Fourier transform is  $\tilde{h}(f) = \mathcal{A}f^{-7/6}\exp[i\Psi(f)]$ . The overall amplitude is the usual GR quantity:  $\mathcal{A} = 30^{-1/2}\pi^{-2/3}\mathcal{M}^{5/6}D_L^{-1}$ , where  $\mathcal{M} = \eta^{3/5}m$  is the chirp mass and  $D_L$  is the luminosity distance. The Fourier phase  $\Psi(f) = \Psi_{\text{GR}}(f) + \delta\Psi(f)$ , where  $\Psi_{\text{GR}}(f)$  is the GR result [37–39], while

$$\delta\Psi(f) = \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}(-10\delta C u^4) \quad (10)$$

is the CS correction. This correction enters at 2PN order, and thus, it is degenerate with the spin-spin PN correction to the GR Fourier phase.

Such a modified gravitational waveform is naturally contained in the parameterized post-Einsteinian (ppE) framework [40], aimed at proposing waveforms that model-independently deviate from GR. The simplest ppE Fourier phase is  $\Psi_{\text{PPE}} = \Psi_{\text{GR}} + \beta_{\text{PPE}}(\pi\mathcal{M}f)^{b_{\text{PPE}}}$ , which can be mapped to Eq. (10) through the ppE parameters

$$\beta_{\text{PPE}} = -\frac{15}{64}\delta C \eta^{-4/5}, \quad b_{\text{PPE}} = -\frac{1}{3}. \quad (11)$$

The CS correction depends on the coefficient  $\delta C$ , which in turn depends on a complicated expression [Eq. (9)]. Such an expression simplifies greatly when the spin angular momentum is aligned or counter-aligned with the orbital angular momentum:

$$\begin{aligned}
\delta C = & -\frac{309845}{553728}\zeta\frac{1}{\eta^2}\left[\left(1-\frac{47953}{61969}\eta\right)\chi_s^2\right. \\
& \left.+\left(1-\frac{199923}{61969}\eta\right)\chi_a^2-2\delta_m\chi_s\chi_a\right]. \quad (12)
\end{aligned}$$

This result seems divergent in the extreme mass-ratio limit  $\eta \ll 1$ :  $\delta C \propto \zeta(\chi_s - \chi_a)^2/\eta^2$ . However, if we take the limit  $m_1 \rightarrow \infty$  while keeping  $m_2$  and  $u$  fixed,  $\delta C$  actually scales as  $m_1^{-2}m_2^{-2}$  [29]. Astrophysical BHs have a theoretical lower mass bound of  $3M_\odot$ , but one might want to consider a primordial BH with arbitrarily small mass. If we take  $m_2 \rightarrow 0$ ,  $\delta C$  does diverge, but this is beyond the small-coupling approximation that we have used, as we explain below.

*Validity of Approximations.*— We have here employed several approximations. First, we assumed a *small coupling*, i.e.  $\xi \ll M^4$  where  $M$  is the smallest mass scale

of the problem. This approximation is required because Eq. (1) represents an *effective theory*; an expansion to quadratic order in the curvature. Higher curvature terms are being neglected and these would enter with higher powers of  $\zeta' \equiv \xi/M^4$ . Therefore, the field equations derived from Eq. (1) are also only valid to linear order in  $\zeta'$ . It is obvious that if one takes one of the BH masses to zero, the small coupling approximation is violated. Second, an exact, closed-form solution that represents a spinning BH in dynamical CS gravity is currently known only to  $\mathcal{O}(\chi_A^2)$  in a  $\chi_A \ll 1$  expansion. Therefore, the potential energy in Eq. (1) is formally only valid to  $\mathcal{O}(\chi_A^2)$ , and thus, so is the waveform computed above.

Since scalar radiation is caused by scalar dipole charges, we can estimate the accuracy of the slow-rotation expansion by computing the dipole charge to all orders in  $\chi$  and then comparing this to an expression truncated to  $\mathcal{O}(\chi^2)$ . The dipole charge  $\mu_A$  is defined as the asymptotic coefficient of the  $r^{-2}\cos\theta$  term in the solution for  $\vartheta$ , when considering an isolated spinning BH in dynamical CS gravity. The variation of the action with respect to  $\vartheta$  [30, 41] leads to the  $\vartheta$  equation of motion, as given in Eqs. (10) and (25) of [31]. Employing a multipolar decomposition, we can solve this equation at dipole ( $\ell = 1$ ) harmonic order using Green’s function methods and exactly obtain the scalar dipole charge as

$$\mu_A^{(\text{full})} = \frac{\alpha}{\beta} \frac{2 + 2\chi_A^4 - 2\sqrt{1 - \chi_A^2} - \chi_A^2(3 - 2\sqrt{1 - \chi_A^2})}{2\chi_A^3}. \quad (13)$$

For  $|\chi_A| < 0.8$ , the difference between  $\mu_A^{(\text{full})}$  and its truncated expansion at  $\mathcal{O}(\chi^2)$  is always less than 10%.

*Future Constraints with GW Observations.*—Let us assume that a GW observation has been made and found consistent with GR. One can then ask how large  $\zeta$  can be to be consistent with such an observation, thus placing a constraint on  $|\xi^{1/4}|$ . One can estimate such a constraint through a Fisher analysis [34]. For sufficiently high SNR, the accuracy to which a given parameter  $\theta^a$  can be measured can be estimated via  $(\Delta\theta^a) = \sqrt{(\Gamma^{-1})^{aa}}$  (no summation implied), where

$$\Gamma_{ab} \equiv 4\text{Re} \int_{f_{\min}}^{f_{\max}} \frac{\partial_a \tilde{h}(f) \partial_b \tilde{h}(f)}{S_n(f)} df, \quad (14)$$

is the Fisher matrix, partial derivatives are with respect to  $\theta^a$ , and  $S_n(f)$  is the noise spectral density for Adv. LIGO (also for Adv. VIRGO and KAGRA) [42], ET [43], LISA [44] and DECIGO/BBO [45]. The limits of integration  $f_{\min}$  and  $f_{\max}$  will be here taken to be

$$f_{\min} = \max(f_{\text{low}}, f_{1\text{yr}}), \quad f_{\max} = \min(f_{\text{high}}, f_{\text{end}}), \quad (15)$$

where  $f_{\text{low}}$  and  $f_{\text{high}}$  are lower and higher cutoff frequencies of a given detector, respectively, while  $f_{1\text{yr}}$  is the GW frequency 1 year prior to coalescence and  $f_{\text{end}}$  is the frequency at the innermost stable circular orbit which

can be obtained by solving  $\hat{C}_0 = 0$  where  $\hat{C}_0$  is given in Eq. (1.5) of [46].

Let us first concentrate on the spin-aligned and spin anti-aligned cases. Such a waveform has eight parameters:  $\{\theta^a\} = \{\ln \mathcal{M}, \ln \eta, \chi_s, \chi_a, t_c, \phi_c, D_L, \zeta\}$ . We here choose the fiducial values  $t_c = \phi_c = \zeta = 0$  and impose the prior  $\chi_A \leq 1$ . ET and DECIGO/BBO should allow us to constrain  $\xi^{1/4} \lesssim \mathcal{O}(10 - 100)\text{km}$ . As expected, the constraint scales as the smallest length scale of the target system (i.e. the horizon size of the smaller BH), and hence it is of  $\mathcal{O}(M)$ . Due to this reason, LISA can only place  $\xi^{1/4} \lesssim \mathcal{O}(10^5 - 10^6)\text{km}$ . Second-generation ground-based detectors are not sensitive enough to measure  $\zeta$  with non-precessing binaries, within the small coupling approximation, due to degeneracies between  $\zeta$  and  $\chi_A$ .

Let us now consider spin precessing binaries. In such systems, all momenta precess around the total angular momentum, thus breaking certain degeneracies between the CS corrections and the spin-spin GR couplings. The CS correction to the precession equations will also enter at 2PN order relative to GR, but since the effect of spin already enters at 1.5PN order relative to the leading Newtonian term, such CS precession corrections can be neglected. Therefore, it would suffice to evolve precessing spins with GR evolution equations [47, 48]. To simplify the problem further, instead, we will assume that when spins precess,  $\chi_{1,2}$  become uncorrelated to the other template parameters, which amounts to repeating the calculation of the spin-aligned case, but assuming the spin parameters are known *a priori*. This approximation has been shown to be very accurate, for example when considering GW bounds on the graviton Compton wavelength [48].

The projected bounds on  $\xi^{1/4}$  for systems with precessing spins under this approximation are shown in Fig. 1 as a function of  $(m, \chi_2)$ , with  $(m_1/m_2, \chi_1) = (2, 0.4)$ , and at a fixed luminosity distance of  $D_L = 0.1\text{Gpc}$  for Adv. LIGO and  $D_L = 1\text{Gpc}$  for ET, LISA and DECIGO/BBO. The colored contours show constraints that satisfy  $\Delta\zeta' < 1$  [where the smallest length scale of the system is taken to be the horizon of the smaller BH, i.e.  $M = m_2(1 + \sqrt{1 - \chi_2^2})$ ], as otherwise the small-coupling approximation is violated. Of course, the regions that satisfy  $\Delta\zeta' < 1$  depend on the choice of  $D_L$ : the colored contours would be larger if one detects a GW from a closer binary and thus with a higher SNR. We have here chosen a rather small  $D_L$  (especially for Adv. LIGO) in view of the expected detection rate. The bounds one could place on  $\xi^{1/4}$  are now stronger as precession has destroyed correlations that are present in the spin-aligned or anti-aligned case. This time, there are parameter regions where second-generation ground-based detectors, such as Adv. LIGO, Adv. Virgo, KAGRA, as well as ET, could constrain dynamical CS gravity to

$$\xi^{1/4} \lesssim \mathcal{O}(10 - 100)\text{km}. \quad (16)$$

This is six to seven orders of magnitude stronger than current Solar System bounds [16].

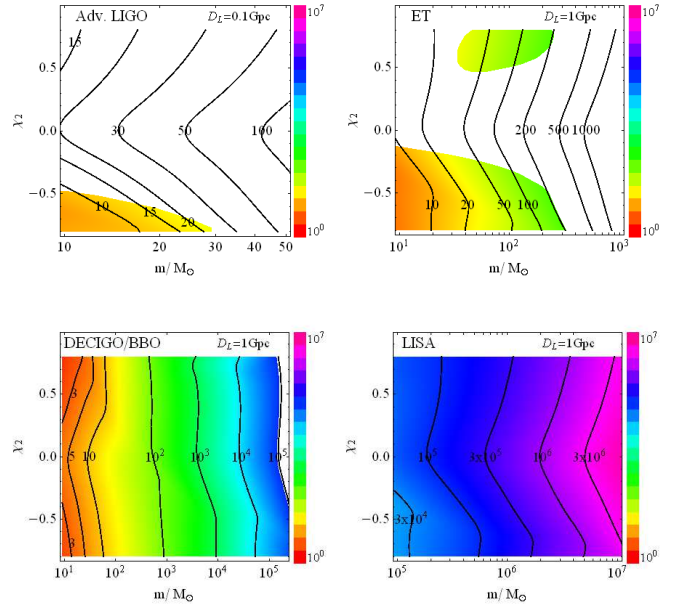


FIG. 1: Projected  $1\sigma$ -constraints on  $\xi^{1/4}$  in km with Adv. LIGO (top left), ET (top right), DECIGO/BBO (bottom left) and LISA (bottom right) at a fixed luminosity distance (shown on the top right of each panel), spin-aligned BH binaries with  $(\chi_s, \chi_a)$  assumed measured,  $m_1/m_2 = 2$  and  $\chi_1 = 0.4$ . This roughly models projected constraints with precessing BH binary observations. The colored contours show the regions of parameter space where the constraints on  $\xi^{1/4}$  also satisfy  $\Delta\zeta' < 1$ , and thus, the small coupling approximation is satisfied at the fiducial luminosity distances chosen.

*Future Work.*— This paper opens the door to several follow-ups. One possibility is to include eccentricity and precessing spins in the inspiral evolutions to estimate how much these effects improve or deteriorate the projected constraints. One could also carry out a Bayesian parameter estimation and a model-selection study to estimate projected constraints for signals with low SNR [42].

The projected constraints presented here rely on several approximations, but all systems analyzed were chosen such that these constraints are rather conservative. Perhaps the dominant assumption is the truncation of the inspiral at the ISCO. Indeed, CS corrections become larger as the binary shrinks, so if one could integrate all the way up to the light-ring and merger, the projected constraints would be much stronger. For this, one would have to develop an effective-one-body resummation of the waveform, which would in turn require the numerical simulation of BH binaries in dynamical CS gravity.

One could also investigate highly spinning BHs and other neutron star binaries. We have succeeded in obtaining the dipole charge for arbitrarily rapidly rotating



BHs, but the waveform also depends on the deviation of the BH metric's quadrupole moment. A study of binary neutron stars in dynamical CS gravity might allow immediate constraints through binary pulsar observations.

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